12.4 Videos Guide

12.4a

- The cross product: Let $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$
  - $\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = (a_2 b_3 - a_3 b_2)\mathbf{i} - (a_1 b_3 - a_3 b_1)\mathbf{j} + (a_1 b_2 - a_2 b_1)\mathbf{k}$
  - $\mathbf{a} \times \mathbf{b}$ is orthogonal to both $\mathbf{a}$ and $\mathbf{b}$

Exercise:

- Find the cross product $\mathbf{a} \times \mathbf{b}$ for $\mathbf{a} = \langle 2, -1, 5 \rangle$ and $\mathbf{b} = \langle -4, 3, 8 \rangle$.

12.4b

- Geometric applications and interpretations of the cross product
  - $|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}| \sin \theta$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$
  - If $\mathbf{a}$ and $\mathbf{b}$ are parallel, then $\mathbf{a} \times \mathbf{b} = \mathbf{0}$, the zero vector
  - $|\mathbf{a} \times \mathbf{b}|$ gives the area of a parallelogram formed by $\mathbf{a}$ and $\mathbf{b}$
  - Scalar triple product: $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$
    - $|\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})|$ gives the volume of a parallelepiped with edges $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$

Exercises:

12.4c

- Find the area of the parallelogram with vertices $P(1, 0, 2)$, $Q(3, 3, 3)$, $R(7, 5, 8)$, and $S(5, 2, 7)$.

12.4d

- Find a nonzero vector orthogonal to the plane through the points $P(0, 0, -3)$, $Q(4, 2, 0)$, and $R(3, 3, 1)$, and (b) find the area of triangle PQR.